

AD-A229 340

NATO Advanced Study Institute  
on Waveguide Optoelectronics

Glasgow, 1990

②  
DTIC FILE COPY

DESIGN AND MODELLING OF PASSIVE AND ACTIVE  
OPTICAL WAVEGUIDE DEVICES

Roel BAETS

Laboratory of Electromagnetism and Acoustics (LEA)

University of Gent, Belgium



R&D 6414-EE-02  
DATA 45-90-M-0130  
USARD+SG(uk)

OUTLINE

① MODES IN WAVEGUIDES

DISTRIBUTION STATEMENT A  
Approved for public release;  
Distribution Unlimited

② LONGITUDINALLY VARIABLE STRUCTURES

③ COUPLED MODE THEORY

- COUPLERS
- STRUCTURES WITH GRATINGS

④ COUPLING LIGHT INTO WAVEGUIDES

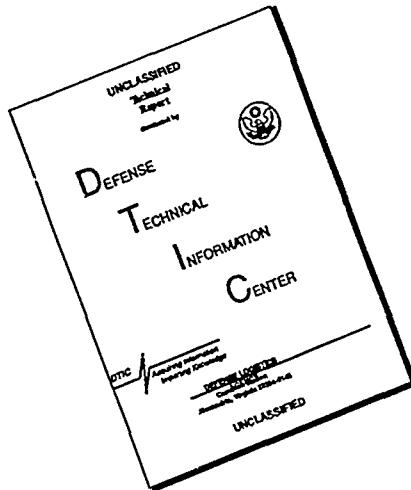
DTIC  
ELECTED  
NOV 20 1990  
S B D  
60

⑤ WAVEGUIDES IN LASER CAVITIES

90 11 20 061

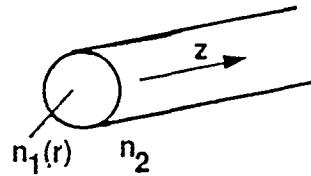
⑥ SOME GENERALITIES ABOUT WAVEGUIDE  
MODELING AND DESIGN

# DISCLAIMER NOTICE



THIS DOCUMENT IS BEST  
QUALITY AVAILABLE. THE COPY  
FURNISHED TO DTIC CONTAINED  
A SIGNIFICANT NUMBER OF  
PAGES WHICH DO NOT  
REPRODUCE LEGIBLY.

## WAVE PROPAGATION MODEL DEFINITION OF PROBLEM



$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} \quad \nabla \times \bar{H} = - \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = 0 \quad \nabla \cdot \bar{B} = 0$$

$$\bar{D} = \epsilon_0 n^2 \bar{E} \quad \bar{B} = \mu_0 \bar{H}$$

Find solutions of the form (modes) :

$$\bar{E} = \bar{E}(r, \phi) \exp[j(\omega t - \beta z)]$$

$$\bar{H} = \bar{H}(r, \phi) \exp[j(\omega t - \beta z)]$$

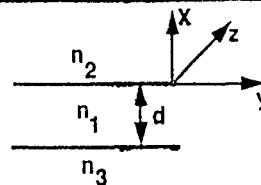
Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
per Form 50	
By Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

## WAVE PROPAGATION MODEL SLAB WAVEGUIDE

Two independent sets of solutions :

$$E_y, H_x, H_z \quad (\text{TE})$$

$$H_y, E_x, E_z \quad (\text{TM})$$



$$\text{TE} : \frac{d^2 E_y}{dx^2} + (n^2 k^2 - \beta^2) E_y = 0 \quad (k = \frac{\omega}{c})$$

$$\left\{ \begin{array}{ll} E_y = A e^{-\delta x} & x \geq 0 \\ E_y = A \cos \kappa x + B \sin \kappa x & 0 \leq x \leq -d \\ E_y = (A \cos \kappa d - B \sin \kappa d) e^{\gamma(x+d)} & x \leq -d \end{array} \right.$$

$$\text{with } \delta = \sqrt{\beta^2 - n_2^2 k^2} \quad \kappa = \sqrt{n_1^2 k^2 - \beta^2} \quad \gamma = \sqrt{\beta^2 - n_3^2 k^2}$$

## WAVE PROPAGATION MODEL SLAB WAVEGUIDE

Continuity of  $E_y$  and  $H_z$  ( $\sim \frac{\partial E_y}{\partial x}$ ) at interfaces

→ eigenvalue equation

$$\operatorname{tg} \kappa d = \frac{\kappa_x (\gamma + \delta)}{\kappa^2 - \gamma \delta} \rightarrow \text{discrete number of solutions for } \beta$$

$$\rightarrow \beta(\lambda, n_1, n_2, n_3, d)$$

## WAVE PROPAGATION MODEL SLAB WAVEGUIDE

### NORMALISATION

Waveguide characterized by

$$v = k_0 d \sqrt{n_1^2 - n_2^2}$$

v-number  
(normalized frequency or thickness)

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}$$

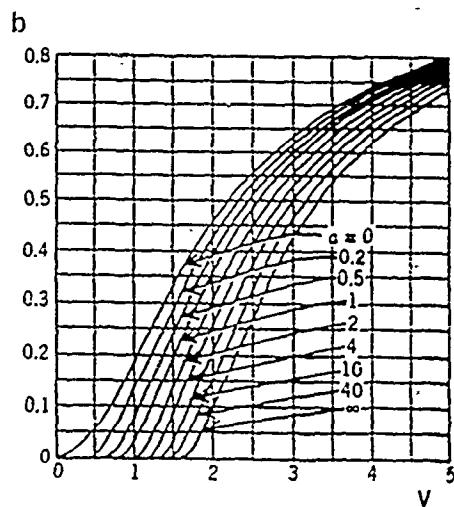
asymmetry factor

Mode characterized by

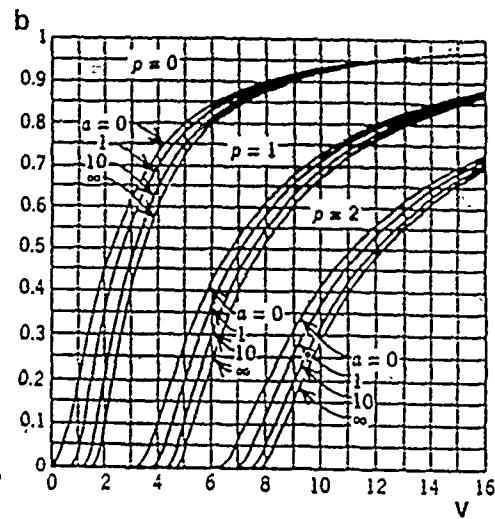
$$\beta = k_0 n_{\text{eff}}$$

$$b = \frac{n_{\text{eff}}^2 - n_2^2}{n_1^2 - n_2^2}$$

## WAVE PROPAGATION MODEL SLAB WAVEGUIDE



FUNDAMENTAL TE MODE



3 LOWEST ORDER TE MODES

### CONFINEMENT FACTOR $\Gamma$

$\Psi(x)$

$n_2$

$n_1$

$n_2$

$\Gamma = \frac{\int_{\text{core}} |\Psi|^2 dx}{\int_{-\infty} |\Psi|^2 dx}$

If the core layer shows gain  $g$  [cm<sup>-1</sup>]  
then the modal gain is  $\Gamma_g$

$$n(x, y) = n_0(x) + \Delta n(x, y)$$

$$\nabla_T^2 E_y + (k_0^2 n^2(x, y) - \beta^2) \Xi_y = 0$$

Assumption:  $\Xi_y(x, y) = F(x, y) \cdot G(y)$  where fast variations along  $y$  are taken up in  $G(y)$

$$\rightarrow \frac{\partial F}{\partial y} = 0$$

$$\rightarrow \underbrace{\frac{1}{G} \frac{d^2 G}{dy^2}}_{\text{function of } y} + \underbrace{\frac{1}{F} \frac{\partial^2 F}{\partial x^2} + k_0^2 n^2 - \beta^2}_{\text{function of } x \text{ and } y} = 0$$

$$\rightarrow \frac{1}{F} \frac{\partial^2 F}{\partial x^2} + k_0^2 n^2 = f^2(y)$$

$$\frac{1}{G} \frac{d^2 G}{dy^2} - \beta^2 = -f^2(y)$$

$$f \triangleq k_0 n_{\text{eff}}(y)$$

Approximation of  $n_{\text{eff}}(y)$  by perturbation method

$$n(x, y) = n_0(x) + \Delta n(x, y)$$

$$F_0(x) \text{ satisfies: } \frac{d^2 F_0}{dx^2} + k_0^2 (n_0^2 - n_{\text{eff}, 0}^2) F_0 = 0 \quad (1)$$

$$F(x, y) \text{ satisfies: } \frac{\partial^2 F}{\partial x^2} + k_0^2 (n^2 - n_{\text{eff}}^2) F = 0 \quad (2)$$

$$n^2 \approx n_0^2 + 2 \Delta n n_0$$

$$\begin{aligned} \int_{x \text{-axis}} (1) F - (2) F_0 &\rightarrow (n_{\text{eff}}^2(y) - n_{\text{eff}, 0}^2) \int_{-\infty}^{\infty} F_0 F dx = \int_{-\infty}^{\infty} 2 \Delta n n_0 F F_0 dx \\ &\rightarrow n_{\text{eff}}^2(y) = n_{\text{eff}, 0}^2 + \frac{\int_{-\infty}^{\infty} 2 \Delta n n_0 F F_0 dx}{\int_{-\infty}^{\infty} F_0 F dx} \\ &\approx n_{\text{eff}, 0}^2 + 2 \frac{\int_{-\infty}^{\infty} \Delta n n_0 F_0^2 dx}{\int_{-\infty}^{\infty} F_0^2 dx} \end{aligned}$$

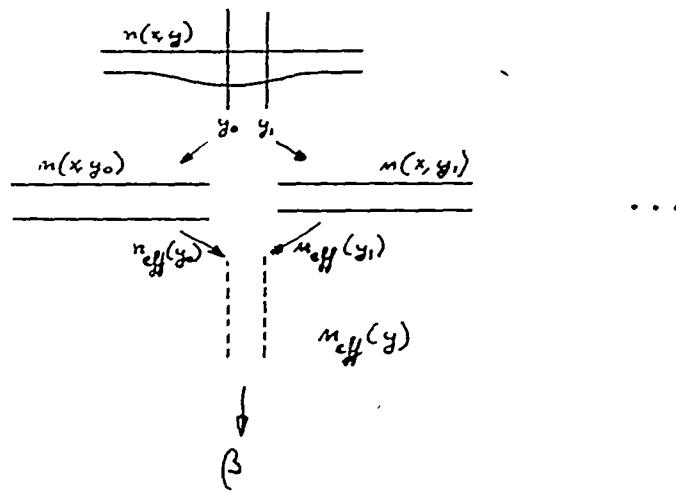
## EFFECTIVE INDEX METHOD

W6

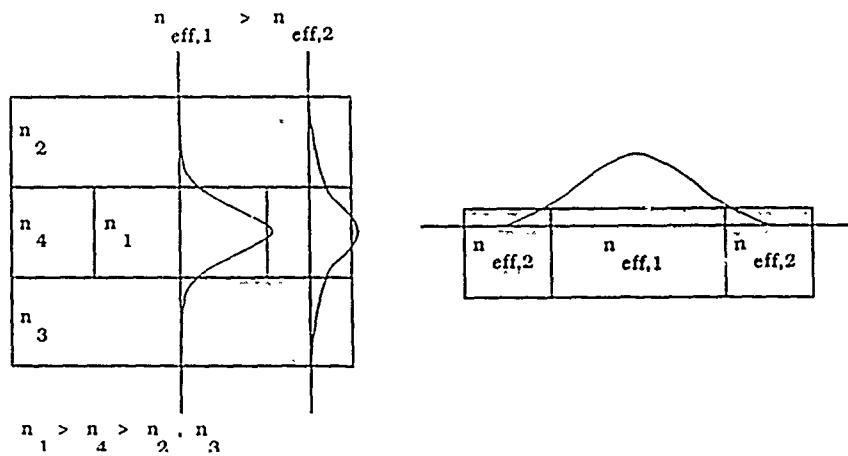
$$\rightarrow \begin{cases} \frac{\partial^2 F}{\partial x^2} + (k_0^2 n^2(x, y) - k_0^2 n_{eff}^2(y)) F = 0 \\ \frac{d^2 G}{dy^2} + (k_0^2 n_{eff}^2(y) - \beta^2) G = 0 \end{cases}$$

1D wave equation  
(to be solved for each  $y$ )

1D wave equation  
(to be solved once)

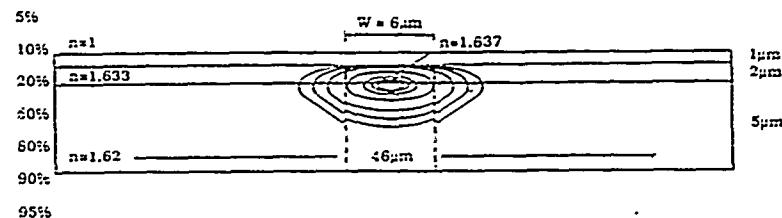


## EFFECTIVE INDEX METHOD

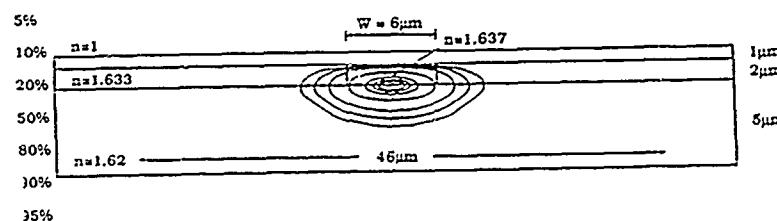


## FUNDAMENTAL WAVEGUIDE MODE

### A. EFFECTIVE INDEX METHOD



### B. FINITE DIFFERENCE METHOD



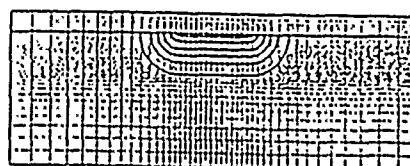
## FINITE DIFFERENCE METHOD

### BASIC EQUATION:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + (k_0^2 \epsilon_r - \beta^2) \phi = 0 \quad (1)$$

\* A finite cross section is defined by enclosing the guide in a rectangular box, where  $\phi = 0$  on the side walls

\* In this box a graded mesh is defined



## FINITE DIFFERENCE METHOD

Continuity conditions :

$$\left. \begin{array}{l} \phi \\ \frac{\partial \phi}{\partial n} \end{array} \right\} \text{continuous at boundary between } S_v$$

Substitution of the discreted form of  $\nabla^2 \phi$  and the continuity conditions into (1) leads to :

$$\begin{aligned} & \frac{2}{w(e+w)} \phi_w + \frac{2}{s(n+s)} \phi_s - \left( \frac{2}{w(e+w)} + \frac{2}{e(e+w)} + \frac{2}{s(n+s)} \right. \\ & \left. \frac{2}{n(n+s)} \right) \phi_P + \frac{2}{e(e+w)} \phi_E + \frac{2}{n(n+s)} \phi_N \\ & + k_0^2 \frac{wn\varepsilon_1 + ws\varepsilon_2 + es\varepsilon_3 + en\varepsilon_4}{(e+w)(n+s)} \phi_P - \beta^2 \phi_P = 0 \end{aligned} \quad (5)$$

## FINITE DIFFERENCE METHOD

Equation (5) holds for each node point P. The resultant eigenvalue equation is of the form

$$[ [A] - \beta^2 [U] ] [X] = 0 \quad (6)$$

with

$$[X] = [\phi_1, \phi_2, \dots, \phi_{NTOT}]^T \quad (7)$$

[U] is the unit matrix, and NTOT is the total number of mesh points. The matrix [A] is a real, but generally not symmetric, sparse matrix. Eigenvalues and corresponding eigenvectors of [A] are found by a simultaneous iteration algorithm.

## BEAM PROPAGATION METHOD TWO DIMENSIONAL

Problem :

Calculation of the propagation of a given input field  $E_0(x,z)$  through a medium with a refractive index  $n(x,z)$

Assumptions :

1. Scalar wave equation

$$\nabla^2 E + k^2 n^2(x,z)E = 0 \quad (1)$$

## BEAM PROPAGATION METHOD TWO DIMENSIONAL

2. Refractive index variation can be written as :

$$n(x,z) = n_0(x) + \Delta n(x,z) \quad (2)$$

where  $\Delta n \ll n_0$  and  $n_0(x)$  chosen so that the solutions of :

$$\nabla^2 \Phi + k^2 n_0^2(x) \Phi = 0 \quad (3)$$

are known eigenfunctions :

$$\Phi_n(x) e^{-ik_n z} \quad (4)$$

In practice.  $n_0 = \text{constant}$

## BEAM PROPAGATION METHOD TWO DIMENSIONAL

3. Neglect the influence of the reflected fields on the forward propagating beam :

- no large abrupt change of  $n(x,z)$  as a function of  $z$
- no periodic reflections that add up coherently

This assumption yields for a field  $\varepsilon(x,z)$  propagating in  $n_0$

$$\varepsilon = \sum_{n=1}^{\infty} B_n \Phi_n e^{ik n z} \quad (5)$$

Boundary value problem transformed into an initial value problem  
 $\Rightarrow$  a stepwise solution feasible

4 Paraxial fields

## BEAM PROPAGATION METHOD TWO DIMENSIONAL

Assume

$$E(x, z_0 + \Delta z) = \varepsilon(x, z_0 + \Delta z) \cdot e^{\Gamma} \quad (7)$$

where

$$\varepsilon(x, z_0 + \Delta z)$$

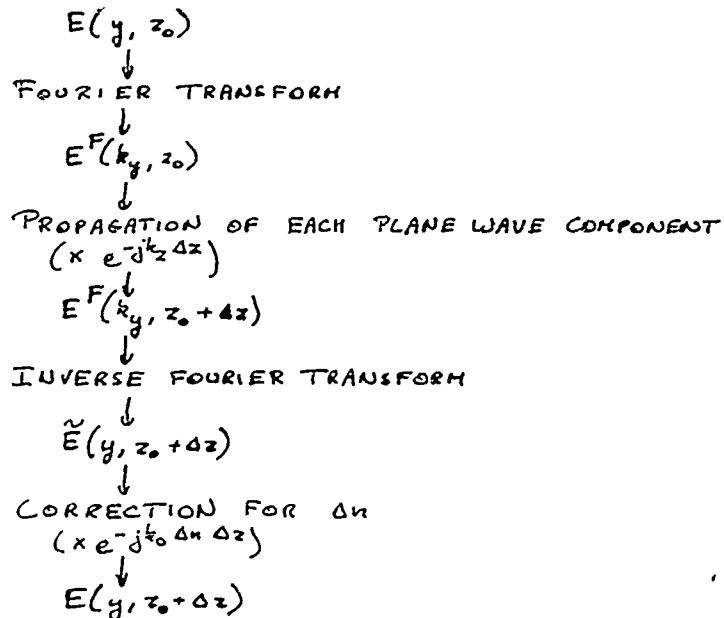
is propagated in the homogeneous medium  $n_0$ ,  
 $e^{\Gamma}$  is a correction factor due to  $\Delta n(x,z)$

$\varepsilon(x,z)$  satisfies

$$\nabla^2 \varepsilon + k^2 n_0^2 \varepsilon = 0 \quad (8)$$

## BPM

### ALGORITHM FOR ONE PROPAGATION STEP



### BOUNDARY CONDITIONS

FFT is

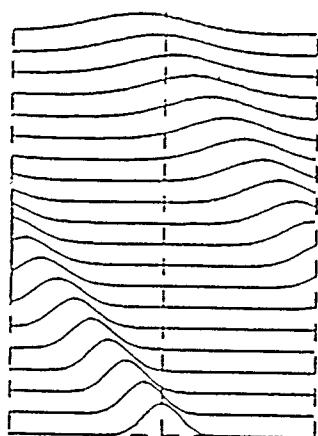
- discrete in both  $x$  and  $k_x$  space
- limited window in both dimensions

FFT is F.T. of the periodic extension of the field

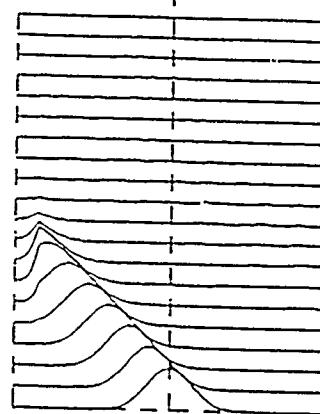
Radiation condition is simulated by absorbing region  
at the edges.  
( - windowing filter )

## GAUSSIAN BEAM HOMOGENEOUS MEDIUM

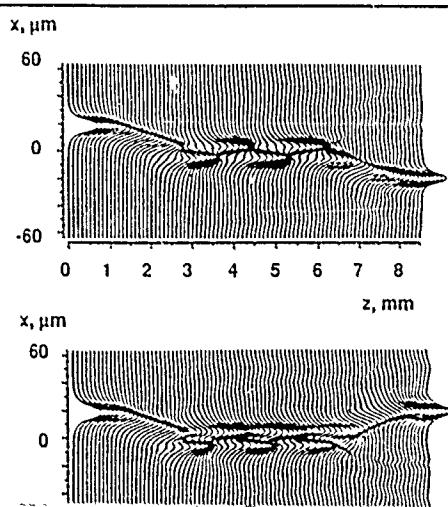
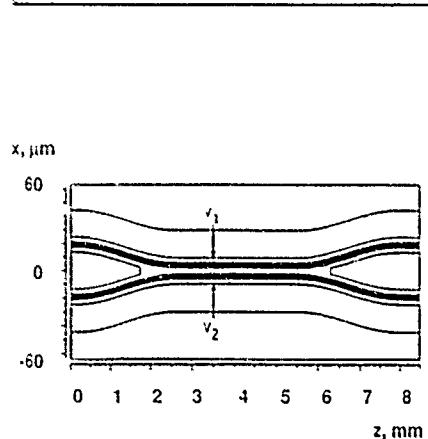
NO ABSORBER

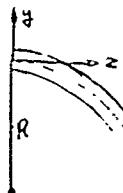


WITH ABSORBER



## BPM EXAMPLE : DIRECTIONAL COUPLER

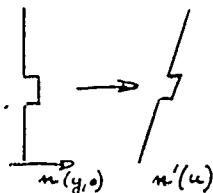




### BENDS

conformal transformation:  $w = u + jw = R \ln \frac{z + j\sqrt{z^2 + R^2}}{R}$

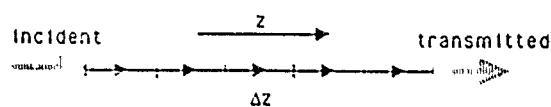
- waveguide in  $(u, v)$  plane is  $v$ -independent
- wave equation becomes new wave equation with  $n'(u) = \exp(u/R) n(u)$   
 $\approx \left(1 + \frac{u}{R}\right) n(u)$  for  $u \ll R$



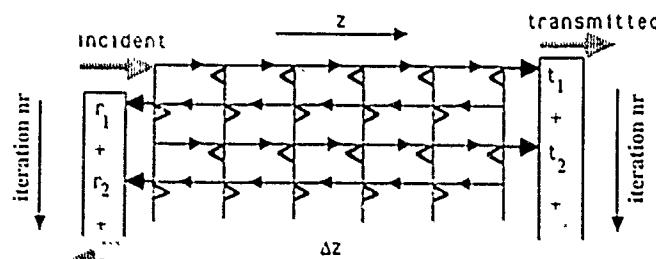
⇒ curved waveguide can be analysed as straight waveguide with modified refractive index profile

### ITERATION PRINCIPLE OF THE UNIDIRECTIONAL & BIDIRECTIONAL BPM

#### UNIDIRECTIONAL BPM

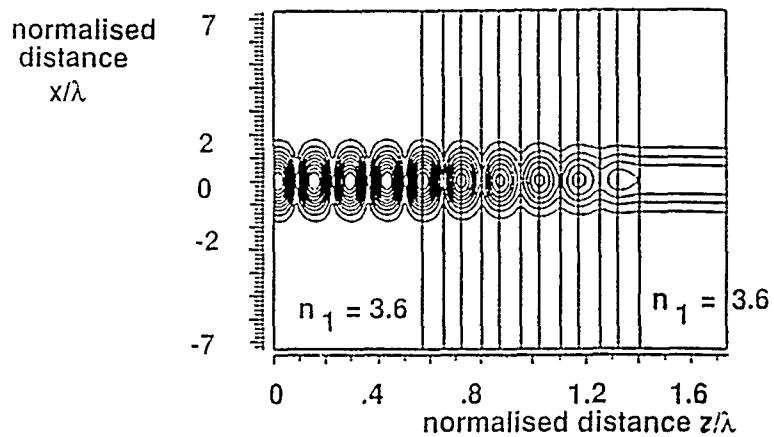


#### BIDIRECTIONAL BPM



## REFLECTIONS OF A GAUSSIAN BEAM INCIDENT ON A BRAGG REFLECTOR

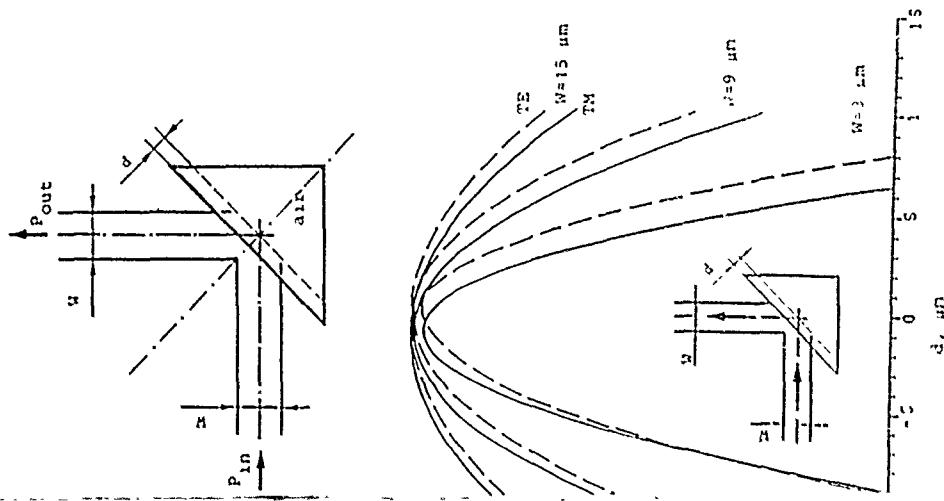
simulated by the bidirectional BPM at  $\lambda = 0.864 \mu\text{m}$



- total field amplitude  $|E(x,z)|$  is normalised to 1
- contour lines with interval 0.1 are plotted

## BS AT 90° CORNER BENDS

### BPM ANALYSIS



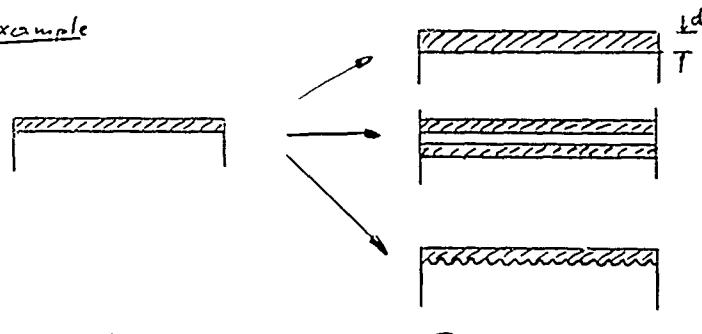
## COUPLED MODE THEORY

W16

ANALYSIS OF DIELECTRIC STRUCTURE WHICH IS A PERTURBATION  
OF A SIMPLER DIELECTRIC STRUCTURE

→ COUPLED MODE THEORY

Example



Unperturbed

Perturbed

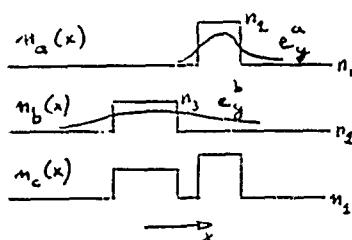
$$\Psi = \sum A_i \Psi_i$$

$$\Psi = \sum A_i(z) \Psi_i$$

$$A_i(z) ?$$

## DIRECTIONAL COUPLER

W20



unperturbed waveguide "a"

unperturbed waveguide "b"

perturbed waveguide (both for "a" and "b")

$$E_y = A(z) e_y^a(x) e^{j(\omega t - \beta_a z)} + B(z) e_y^b(x) e^{j(\omega t - \beta_b z)}$$

$P_{\text{pert}}^a$

$$e^{j\omega t} \epsilon_0 e_y^a(x) A(z) [n_c^2 - n_a^2] e^{j\beta_a z} + e^{j\omega t} \epsilon_0 e_y^b(x) B(z) [n_c^2 - n_b^2] e^{j\beta_b z}$$

$P_{\text{pert}}^b$

$$\rightarrow \left| \frac{dA}{dz} e^{j(\omega t - \beta_a z)} \right| = \frac{j}{\omega} \frac{\partial}{\partial z} \int_{-\infty}^{\infty} [P_{\text{pert}}^a + P_{\text{pert}}^b] e_y^a(x) dx$$

### DIRECTIONAL COUPLER

WL1

$$\begin{cases} \frac{dA}{dz} = -j K_{ab} B e^{+j(\beta_a - \beta_b)z} - j M_a A \\ \frac{dB}{dz} = -j K_{ba} A e^{j(\beta_b - \beta_a)z} - j M_b B \end{cases}$$

with

$$K_{ab} = \frac{\omega \epsilon_0}{4} \int_{-\infty}^{\infty} (n_c^2 - n_a^2) E_y^a E_y^b dx$$

$$M_a = \frac{\omega \epsilon_0}{4} \int_{-\infty}^{\infty} (n_c^2 - n_a^2) (E_y^a)^2 dx$$

Change of variables :  $A = A' e^{-j M_a z}$  and  $B = B' e^{-j M_b z}$

$$\begin{cases} \frac{dA'}{dz} = -j K_{ab} B' e^{j(\beta_a + M_a - \beta_b - M_b)z} \triangleq -j K_{ab} B' e^{2j\delta z} \\ \frac{dB'}{dz} = -j K_{ba} A' e^{j(\beta_b + M_b - \beta_a - M_a)z} \triangleq -j K_{ba} A' e^{-2j\delta z} \end{cases}$$

$$2\delta = \beta_a + M_a - \beta_b - M_b$$

### DIRECTIONAL COUPLER

WL3

CASE 1 :  $\delta = 0$

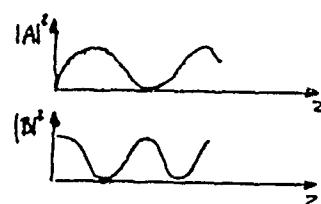
(symmetry)

$$K_{ab} = K_{ba} = \kappa$$

$$|A|^2 = P_0 \sin^2(kz)$$

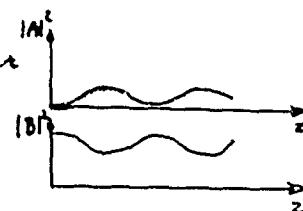
$$|B|^2 = P_0 \cos^2(kz)$$

$$\text{Power transfer length : } L = \frac{\pi}{2k}$$



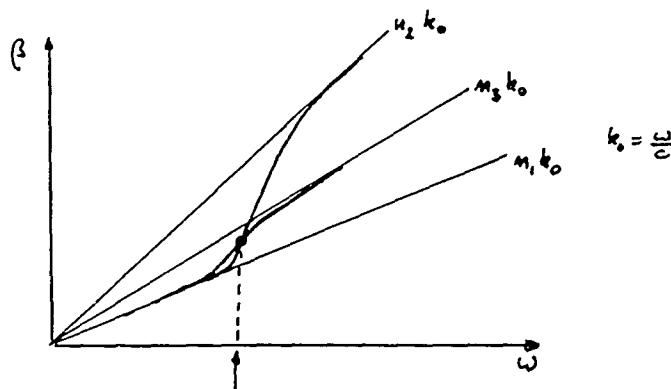
CASE 2:  $\delta \neq 0 \rightarrow N_0$  complete power transfer

$$\frac{|A|_{\max}^2}{P_0} = \frac{|K_{ba}|^2}{K_{ba} K_{ab} + \delta^2}$$



## DIRECTIONAL COUPLER

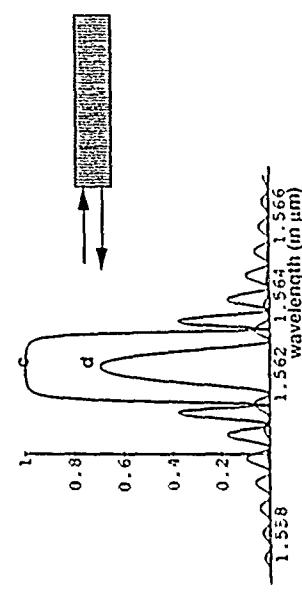
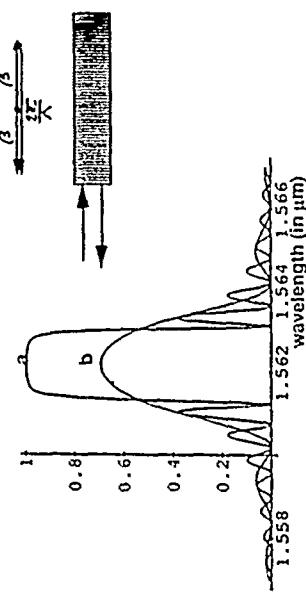
W24



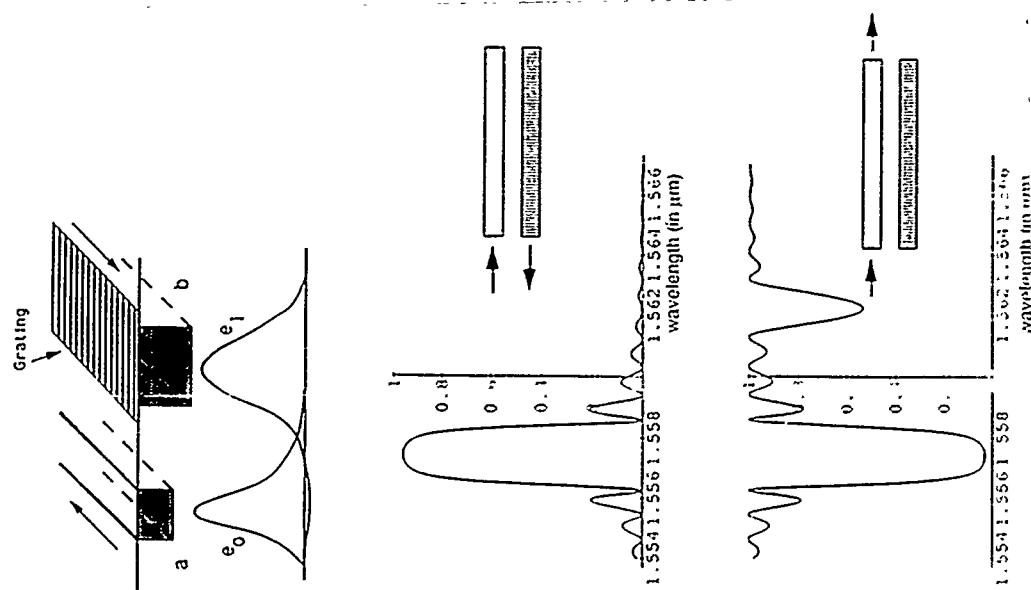
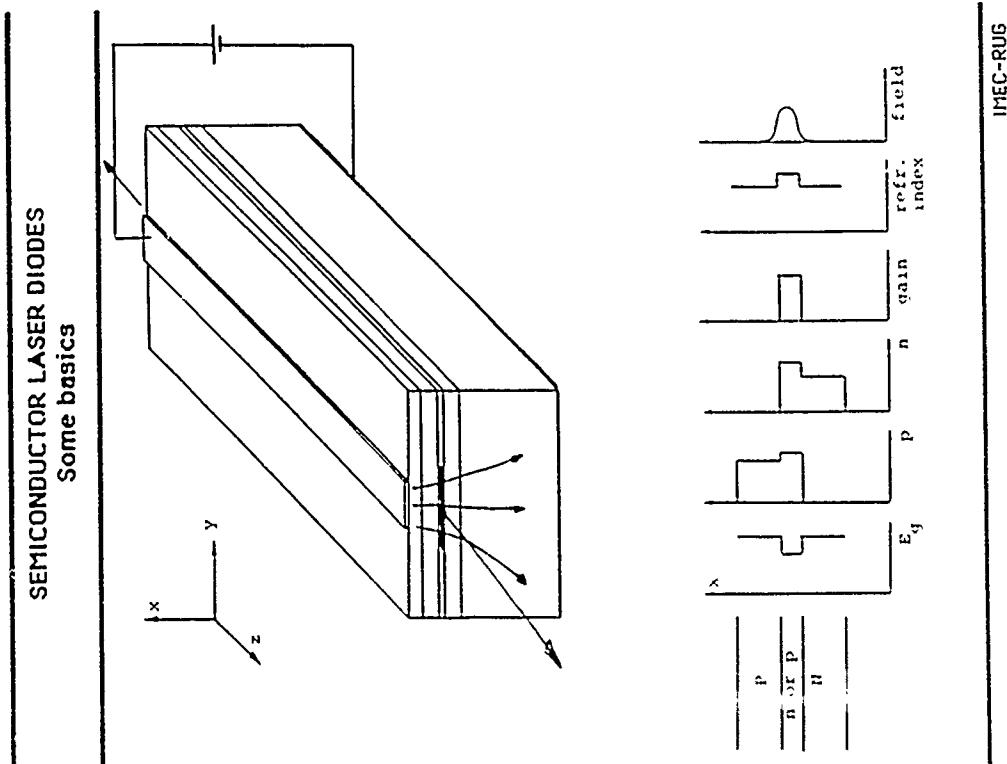
Frequency where coupling is strong

### WAVEGUIDES WITH GRATINGS (BRAGG REFLECTORS) ANALYSED WITH COUPLED MODE THEORY

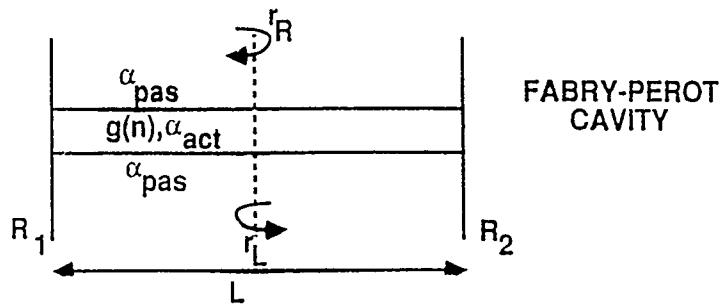
Strong coupling if  $\beta - \frac{2\pi}{\lambda} = -\beta$



- a.  $L = 800 \text{ nm}$ ,  $\lambda = 600 \text{ nm}$
- b.  $L = 200 \text{ nm}$ ,  $\lambda = 600 \text{ nm}$
- c.  $L = 600 \text{ nm}$ ,  $\lambda = 600 \text{ nm}$
- d.  $L = 600 \text{ nm}$ ,  $\lambda = 400 \text{ nm}$



## LASER DIODES Cavity Resonance

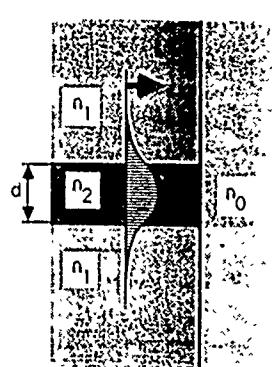
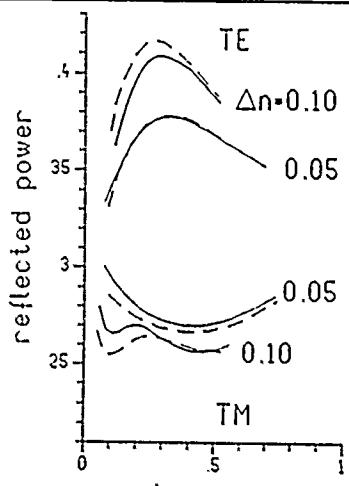


RESONANCE :  $r_L(n, \lambda) \cdot r_R(n, \lambda) = 1$  roundtrip gain

FABRY-PEROT :  $g(n) = \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2}$  (amplitude)

$$\frac{\lambda}{n_r} = \frac{2L}{m}, \text{ m integer (phase)}$$

## BIDIRECTIONAL BPM : REFLECTION FROM A SEMICONDUCTOR LASER FACET

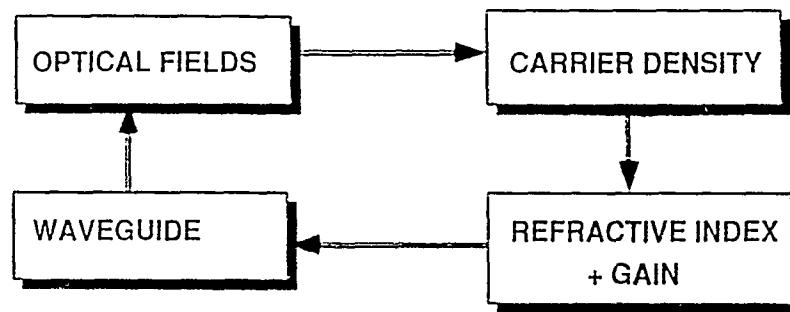


Laser facet configuration

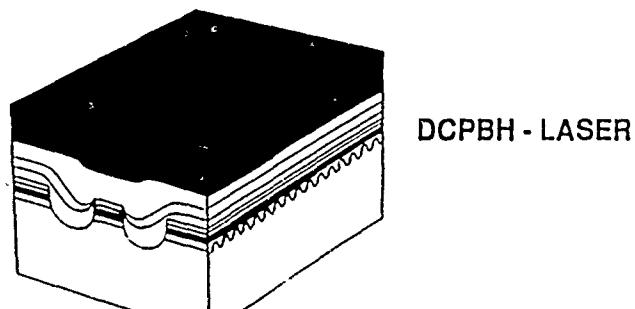
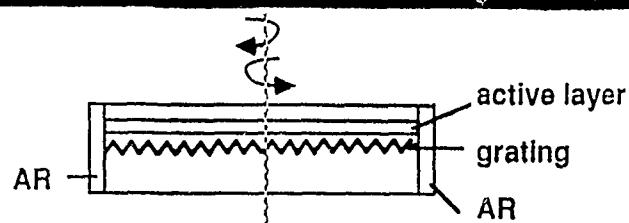
— bidirectional BPM  
..... exact solution

## SELF CONSISTENT MODELLING

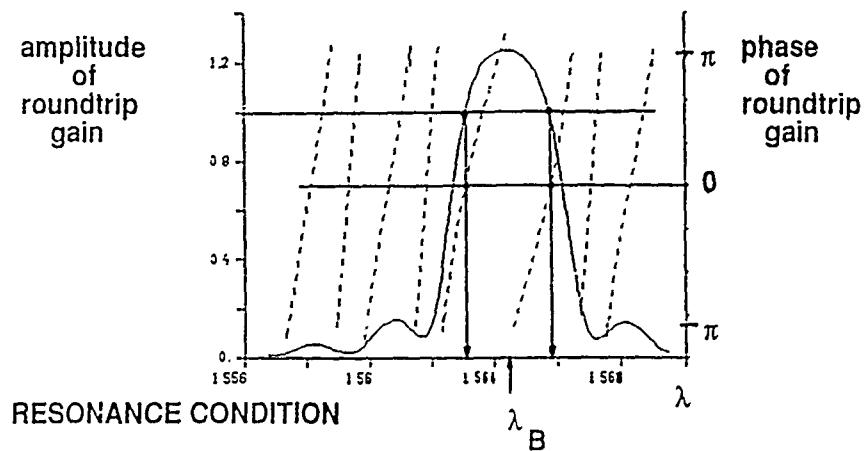
EXAMPLE : ELECTRICAL - OPTICAL INTERACTION



## SLM LASERS DISTRIBUTED FEEDBACK (DFB) LASER



## SLM LASERS DFB LASER



PHASE :- no mode at  $\lambda_B$

- modes symmetrically around  $\lambda_B$

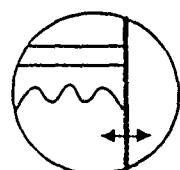
→ two lines in spectrum

## SLM LASERS SINGLE MODE DFB LASERS

### 1. ONE OR TWO NON-AR-COATED FACETS

- ASYMMETRIC LASER
- 2 MODES DO NOT HAVE THE SAME THRESHOLD GAIN
- ONE LINE IN SPECTRUM

PROBLEM WITH FACETS IN DFB LASERS :

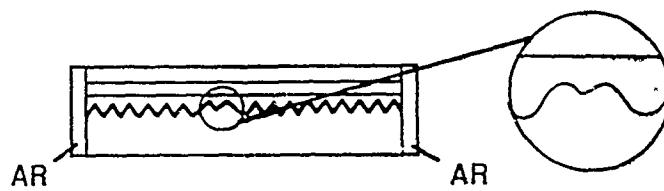


position of facets with respect to grating  
can not be controlled technologically

- some lasers good, some bad
- yield problem

## SLM LASERS SINGLE MODE DFB LASERS

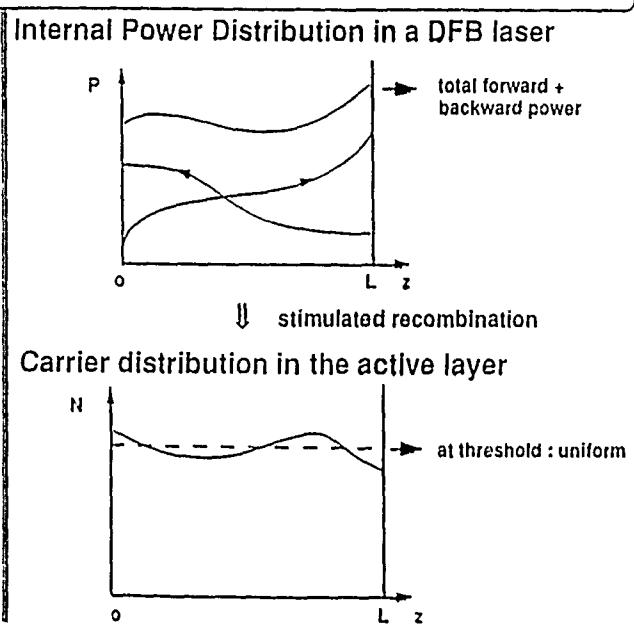
### 2. $\lambda/4$ PHASE SHIFTED LASERS



- PHASE RESONANCE AT  $\lambda_B$
- ONE LASING PEAK AT  $\lambda_B$

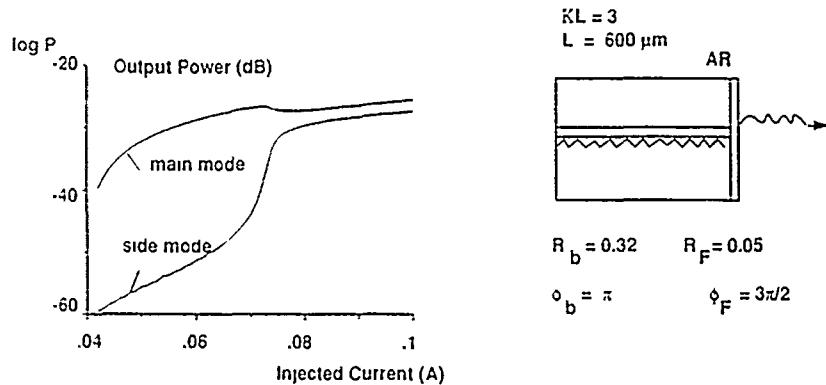
## LONGITUDINAL SPATIAL HOLE BURNING

$P(z)$   
 $\Downarrow$   
 $N(z)$   
 $\Downarrow$   
 $\Delta n_r(z)$   
 $\Downarrow$   
 { Bragg Deviation  
 $\Downarrow$   
 $\frac{2\pi n_{\text{eff}}}{\lambda} (z) - \frac{\pi}{\Lambda}$



## DC ANALYSIS OF DFB LASER

EXAMPLE : UNSTABLE SINGLE MODE BEHAVIOUR DUE TO SPATIAL HOLE BURNING

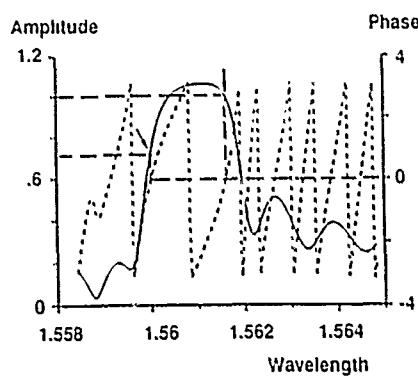


$$I_{th} = 39.56 \text{ mA}, \lambda_{th} = 1.5616, \Delta\alpha L = 0.25$$

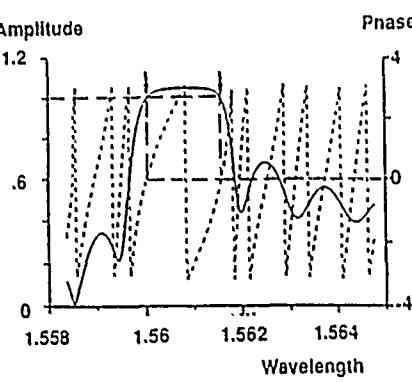
## DC ANALYSIS OF DFB LASER

ROUNDTRIP GAIN OF UNSTABLE DFB LASER

At threshold main mode  
 $I = 39.6 \text{ mA}$



At threshold side mode  
 $I = 70 \text{ mA}$



# SOME CHARACTERISTICS OF OPTICAL WAVEGUIDE MODELS

① Rigorous solution of a waveguide problem is difficult  
↓

People introduce simplifications

BUT: Validity of simplifications not clearly established

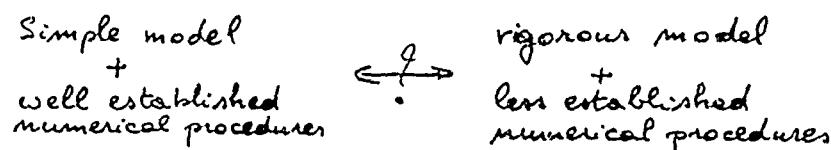
Simplifications very specific to specific structures  
⇒ many different models

② Very few problems can be solved analytically

## Numerical solution methods

BUT:

- numerical solution does not easily allow for design and optimisation within certain parameter space
- numerical solution introduces numerical error



③ It is difficult to compare modelling results with experimental results quantitatively

Because :

- modelling result based on simplified model
- experimental structure not sufficiently well defined

④ All models are limited to single (or few) device level

No higher level "circuit-like" models available.

⑤ Most software implementations are produced by universities

- maintenance + servicing + updating ??

## CONCLUSIONS

- Optical waveguide modelling requires simplification.  
Care is needed to judge the appropriateness of the simplifications
- Modelling is good for
  - demonstrating conceptual ideas theoretically
  - explain experimental observations
  - it is inadequate for accurate design and optimisation

All evolution is needed towards integrated modelling and CAD-tools that contain a number of models with a flexible interface in between them and with sufficient intelligence to protect the user against use beyond the validity range of the models.